Application of Hamilton Circuit or Path in Determining Task Completion Routes in Among Us Games

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***Abstract*—Although it came out in 2018, in recent 2020, Among Us has become a popular video game. The main objective of the game is, as a crew member of a spaceship, the player must traverse the map to do various tasks to repair the spaceship. One might be led to wonder, is there a way of traversing the map so that the task can be done efficiently. According to graph theory, a Hamilton path or circuit is a way to traverse a graph by visiting each node of the graph exactly once. By applying graph theory and using the concept of a Hamilton path or circuit, a way to traverse the map where the player only visits specific locations to do tasks exactly once is possible. By converting the playable maps in among us into graph with nodes representing the key locations where a task may appear, by analysis, one can figure out whether the graph contains a Hamilton path or circuit or not.**

***Keywords*—Among Us, Graph, Hamilton Circuit, Traverse**

I. Introduction

Figure 1. Among Us Logo
Source: Among Us (innersloth.com)

Among us is a multiplayer online game where players must coordinate to finish tasks in a spaceship while trying to find the impostor among them, hence the title (Among Us, 2018). The video game revolves around a team of spaceship engineers and some of the members has been infected by an alien parasite. The game divides the players into 2 categories, the standard crewmate and the alien impostor. The crewmates play the game by doing interactive tasks in which the end goal is to fix the spaceship that they are boarded on and travel to a safe location. The impostor's objective in the game is to kill the other crewmates in order to prevent the repair of the ship. The video game has gained a lot of popularity in the recent times. According to twitchtracker.com, a statistics website for recording viewers on the platform twitch.tv, the video game has gained a peak of over 700.000 viewers in a day as recent as 5 November of 2020. The popularity of the game and the surprising amount of math one can involve in the game itself is one of the most compelling reason for the writer to decide writing this paper.

As mentioned before, the main objective for a player in among us, especially when playing as a crewmate, is to traverse the spaceship, also called the map, to do interactive tasks. In among us, there are several maps one can play in which will be discussed further later in the paper. Inside of these maps, the players who are playing as a crewmate are given tasks to complete on various locations in the map. This prompts the player to move about in the map so as to reach the next task, and the next, so on and so forth. One may start to wonder, is there a way of travelling through the map so that you only visit the location required to do the task only once so as to be as efficient as possible?

Fortunately, the map in among us can be represented as a simple graph. The graph would consist of nodes representing the key places where tasks are required to be finished. The graph would also have edges representing the pathway to access these specific key places. By using graph theory, a graph in which the player would only need to visit each of the key location at least once can be created. The purpose of this research is to construct a graph that can easily be traversed no matter where in the map the required task is located. In particular, this paper would discuss the application of the concept of Hamiltonian graph, a graph where there exists a path that traverse through each of the node exactly once, in order to build the most efficient path one could take when traversing the map of among us.

II. Theoretical Basis

It is important to review the theoretical basis that is related to this paper. In this section, the mathematical basis of graph theory will be presented. This will help in getting a better understanding of how it correlates with the among us video game.

2.1. Graph

According to Munir (2010), a graph is a set of vertex and edges (also referred to as nodes and edges) which represents a relationship between discrete objects (p.356). Graph theory in mathematics refers to the theory concerning these objects. Mathematically, graphs are noted as G = (V, E), representing graph, nodes, and edges respectively. Geometrically graphs can be drawn in a two-dimensional plane using points to represent the vertex/node and lines to represent edges that represents the relations between the points. An example is shown below.

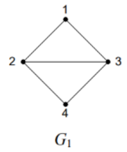

Figure 2. Example of A Graph
Source: Graf (itb.ac.id)

2.2. Types of Graph

There are many types of graph. The types of graph can be viewed from 2 categories. The first category depends on whether the graph contains a loop or multiple edges for a vertex. The second category depends on the graph having direction. According to Munir (2010), the first category is divided again into 2 types (p. 357)

1. Simple graph
2. Un-simple graph

And the second category can be divided into 2 types (p. 358)

1. Undirected graph
2. Directed graph

These are some of the basic terms that are used in identifying what type a graph belongs to. A little summary and explanation of each type will be explored in the subsequent passages.

2.2.1. Simple Graph

A graph that does not contain multiple edges or loops for any of the nodes.

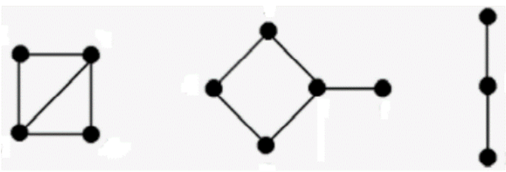

Figure 3. Examples of Simple Graphs
Source: Graf (itb.ac.id)

2.2.2. Un-simple graph

A graph that either contains multiple edges for a single node, or contains a loop, or both.

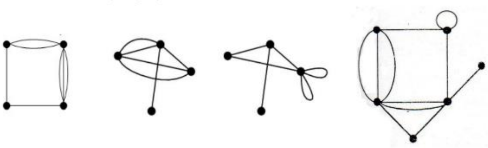

Figure 4. Examples of Un-simple Graphs
Source: Graf (itb.ac.id)

2.2.3. Undirected Graph

A graph that does not have any particular sets of direction for any of the edges.

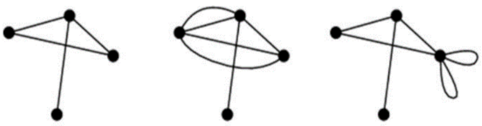

Figure 5. Examples of Undirected Graphs
Source: Graf (itb.ac.id)

2.2.4. Directed Graph

A graph that has directional orientation in all of the edges

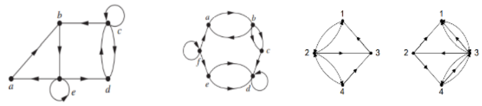

Figure 6. Examples of Directed Graph
Source: Graf (itb.ac.id)

2.3. Graph Terminology

There are several important terminologies related to graphs. It is important to understand each of the terminologies so as to have a clear understanding when discussing about the topic of graphs. The following explanations for the terminologies are summarized and cited from Mr. Rinaldi Munir (2020), a lecturer at Bandung Institute of Technology, on his presentation on the subject of Discrete Mathematics.

2.3.1. Adjacent

Adjacency refers to the relation of 2 nodes. Two nodes are said to be adjacent to each other if there is an edge that directly connects the two nodes together. In the graph G1 below, notice that node 1 and 2 is adjacent, yet the node 1 and 4 is not.

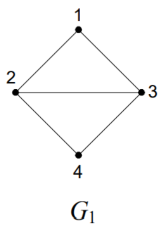

Figure 7. G1 Graph
Source: Graf (itb.ac.id)

2.3.2. Incidency

Incidency refers to the relation of an edge and a node. Mathematically, for a node e= (v_j, v_k), edge e is incident with both v_j and v_k. As shown in the graph G2 below where the edge e_2 is incident with node 1 and 2.

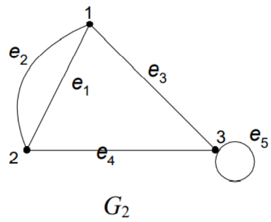

Figure 8. G2 Graph
Source: Graf (itb.ac.id)

2.3.3. Isolated Vertex

An isolated vertex refers to a node/vertex without any incident edges. As shown in the example below, in the graph G3, node 5 is an isolated vertex because it has no edges connected to it.

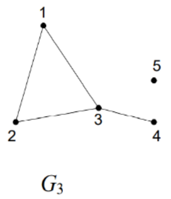

Figure 9. G3 Graph
Source: Graf (itb.ac.id)

2.3.4. Null Graph

A null graph is a graph without any edges. As shown in the example below.

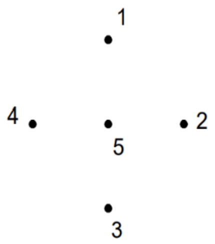

Figure 10. Null Graph Example
Source: Graf (itb.ac.id)

2.3.5. Degree

A degree refers to the number of edges that are connected to a specific node. Mathematically, degrees can be noted as d(v)= x, where d(v) is degree of a vertex, and x is an integer. As shown in the example below, node 3 has a degree of 3, node 2 a degree of 2, and node 5 has a degree of 0 because it is an isolated vertex.

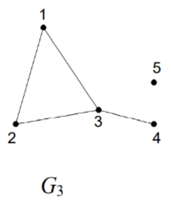

Figure 11. Graph G3
Source: Graf (itb.ac.id)

2.3.6. Path

A path refers to the order of nodes that one travels from a starting node v_0 to an ending node v_n. One can travel through the graph using the edges that connects the many nodes. The length of a path is determined by how many edges one takes when traveling through the graphs. For example, in the graph below, to get from node 1 to 4, we can use the edges (1, 3) and (3, 4) to create a path with a length of 2.

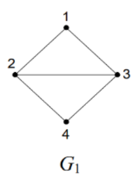

Figure 12. Graph G1
Source: Graf (itb.ac.id)

2.3.7. Circuit

A circuit refers to a path in a graph where the starting node is the same as the ending node. Note that not all graph has a circuit. An example using the previous graph would be the circuit from node 1,2,4,3,1. Circuits are also known as cycles.

2.3.8. Connected

Two nodes v_i and v_f are said to be connected if there is a path from v_i to v_f. A graph can be said to be connected if all of the nodes are connected. The reverse is also true, if there is not a path from one node to the other, the graph is said to be disconnected. Examples for connected and disconnected graphs are shown below.

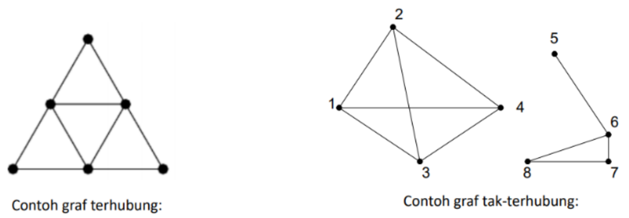

Figure 13. A Connected (Left) and A Disconnected Graph (Right)
Source: Graf (itb.ac.id)

2.3.9. Subgraph

A graph G1 is defined as a subgraph of graph G if all the nodes and edges in G1 is a subset of the nodes and graphs in G. A subgraph also has a component called the complement. The complement, when added to the subgraph, will become the original graph again. Below are shown examples of a graph, subgraph, and its complement.

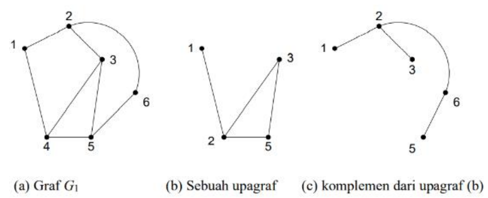

Figure 14. A Graph (Left), subgraph (Center), and Its Complement (Right)
Source: Graf (itb.ac.id)

2.4. Hamilton Circuit

After knowing the basic theory and terminologies of graphs, Hamiltonian Circuit is another important concept that will be used for the analysis of this paper. According to Rosen (2012), a circuit in a graph that passes through every node exactly once is called a Hamilton circuit (p.698). What this means is that for some particular graphs, there can exist a circuit where each of the node is only visited once. In the case of among us, this concept can be used in order to find what circuit or path to take for all tasks to be completed. Below are shown some examples of graphs which contains a Hamilton circuit.

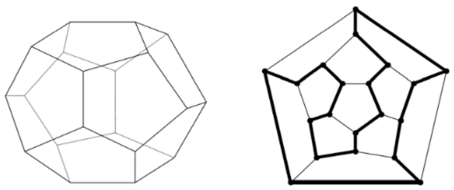

Figure 15. Hamilton's Dodecahedron
Source: [Graf (itb.ac.id)]

According to Munir (2020), a graph with more than 3 nodes can have a Hamilton Circuit if the degree of each of the nodes is at least n/2 where n is the number of nodes. However, there are exception to this rule as shown in the picture below. the following graph has 6 nodes and some of the nodes has less than 3 degrees, yet it still contains a Hamilton circuit.

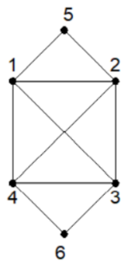

Figure 16. A Graph Containing a Hamilton Circuit
Source: [Graf (itb.ac.id)]

2.5. Hamilton Path

Another important concept to note is the Hamilton path. Similar with the Hamilton circuit, a Hamilton path can be identified within a graph. According to Rosen (2012), a path in a particular graph which goes through each node exactly once is called a Hamilton path. Unlike a Hamilton circuit, a Hamilton path does not necessarily have to return to its starting position after travelling through the graph. This would make it easier to identify in a graph where all the degrees are not equivalent to one another. The following is an example of a graph that contains a Hamilton path but not a Hamilton circuit.

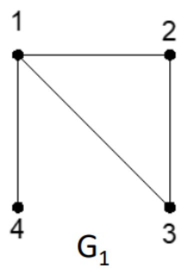

Figure 17. A Graph Only Containing a Hamilton Path
Source: [Graf (itb.ac.id)]

III. AMONG US MAPS

Now armed with the basic knowledge of graph theory, the next step to apply the knowledge is to create graphs out of the Among Us maps. By converting Among Us maps into graphs, it is possible to determine whether the maps have either a Hamilton circuit, Hamilton path, both, or neither. By doing this, one can figure out an effective and efficient way of traversing the maps by only visiting each of the key places once.

As of the time of this paper's writing, there are currently 3 playable maps in Among Us. The maps are themed after space, spaceships, and extraterrestrial exploration. Each of the maps have different key locations where tasks are given and different layouts. The following are pictures and description of each of the playable maps.

Figure 18. The Skeld
Source: Among Us (Video Game)

The first map is called The Skeld. This map is modeled after a spaceship. As shown above, each of the key locations is presented in dark blue, with names on them such as "cafeteria" or "weapons". The pathway of the map is shown by the light blue color. The yellow exclamation mark represents the task in each of the key locations. There is also a player icon to identify the current location of the player. The color scheme and what they represent are the same for each map. In this particular map, the player's starting position will always be in the "Cafeteria". In the graph, the node "Cafeteria" will also be the starting node.

Figure 19. Mira HQ
Source: Among Us (Video Game)

The second map is called Mira HQ. It is modeled after a headquarter building of some sort. The player's starting position in this banner will be the "launch pad" node. However, there is a certain mechanic in the game that makes it so that the player starts at the "cafeteria" node. This only happens when a kill by the impostor is reported or an emergency meeting is called within the game. The emergency meeting and kill reporting are video game mechanics that are used for gameplay which changes the position for all players in the game. This paper will not discuss in detail what the mechanics do. However, it should be noted that the game mechanics are mentioned because of its significance of changing the player's position in the map and subsequently the graph. For this map, the cases before and after a meeting or kill report will be addressed.

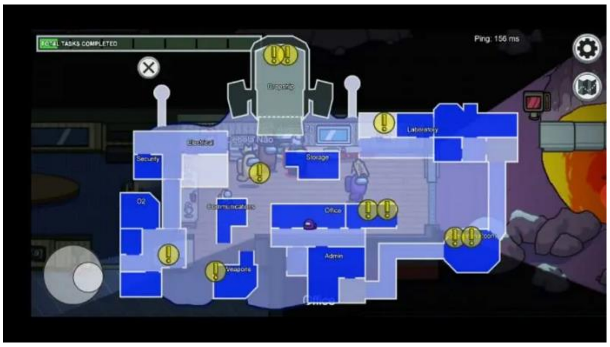

Figure 20. Polus
Source: Among Us (Video Game)

The third and last playable map is called Polus. It is modeled after an extraterrestrial planet. The player's starting position on this map will be on the "dropship" node. Similar to the Mira HQ map, when a kill is reported or an emergency meeting is called, the player's starting position will change to the "office" node. For this map, the cases before and after a kill report or an emergency meeting will also be addressed.

IV. Analysis and Discussion

After knowing the maps that are playable in Among Us, the next task is to convert the maps into graphs. The graphs must be able to represent all the possible path that is to be taken by any player. In this section, all the graphs and whether the graph contain a Hamilton path or circuit based on the starting position, or not, will be thoroughly analyzed. The following explanations are acquired via observation and analysis based on graph theory.

4.1. The Skeld

The following is a graphical representation of The Skeld. The nodes of the graph represent the key locations where tasks may appear and the names of these locations. The edges of the graph represent all the possible routes that a player can take after entering each of the key location. The starting node for this graph, as mentioned before, will be the "Cafeteria node".

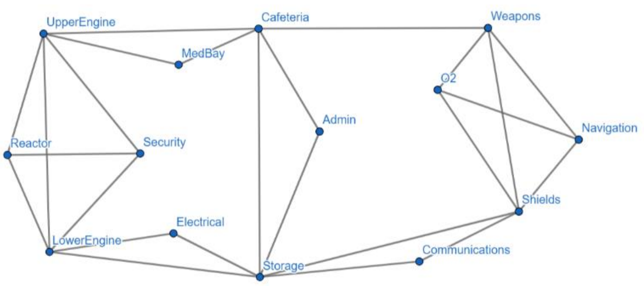

Figure 21. Graphical Representation of The Skeld
Source: Writer

When considering that the starting position or starting node is the cafeteria node, no Hamilton path or Hamilton circuit can be made. This result was reached by trying all the possible combination of paths starting from the "Cafeteria" node. When closely inspected, the "Admin" node maybe the reason for the non-existence of any Hamilton path or circuit. Since the degree of the admin node is 2, both of the edges e_1= (cafeteria, admin) and e_2= (admin, storage) must be used when making the Hamilton path/circuit. With close inspection of the graph, 2 sides can be inferred from the graph. For this analysis, the sides will be called the right and the left side. Below are shown the sides of the graph with blue representing the left side and orange representing the right.

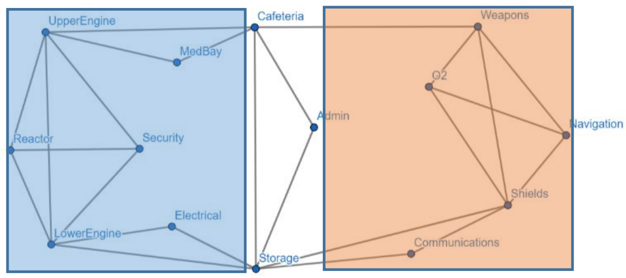

Figure 22. Graph Representation of The Skeld with Sides
Source: Writer

Because of the aforementioned "Admin" node needing both of its edges to be part of the Hamilton path/circuit, there is no possible way for players to traverse from the left side of the graph to the right or vice versa without passing through the center nodes (which could've been previously passed before). Considering this fact, and with the starting node being "Cafeteria", no combinations of path will result in a Hamilton path or circuit.

That said, there is a Hamilton path in this graph. What one must do is consider the starting position as the "Admin" node. When starting from the "Admin" node, the following Hamilton path becomes available.

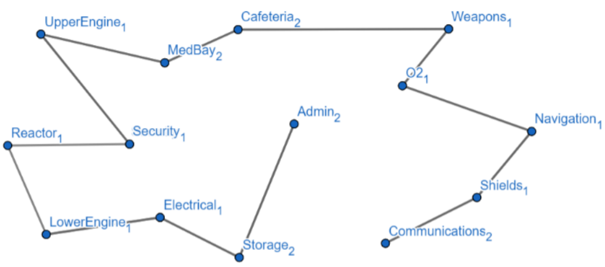

Figure 23. Hamilton Path of The Skeld with Admin as the Starting Node
Source: Writer

This shows that a Hamilton path is still available to access from the graph. When starting the game from the "Cafeteria" node, players can advance to the "Admin" node and take this path in order to traverse all the key location exactly once. This way, even though the starting position of the player in game and the starting position of the graph for the Hamilton path differs, a solution is still available to the player.

4.2. Mira HQ

The following is a graphical representation of the Mira HQ map. Identical to the previous graphical representation, the nodes and edges also represents the key locations and possible routes that the player may take. There are 2 starting points for this graph. The first is the "Launchpad" node and the second is the "Cafeteria" node.

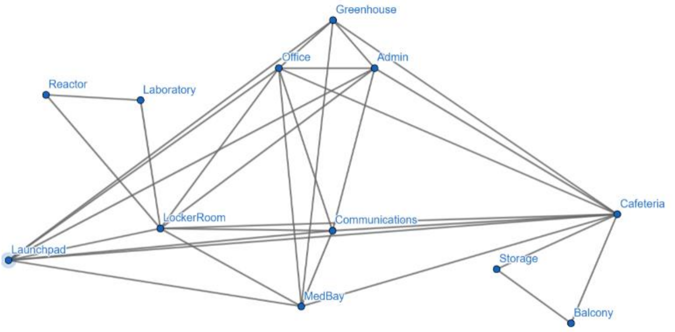

Figure 24. Graph Representation of Mira HQ
Source: Writer

When considering the starting position as the "Launchpad" node, no Hamilton path or Hamilton circuit can be made. This result was reached by trying all the possible combination of paths starting from the "Launchpad" node. The equivalent is also true when starting in the "Cafeteria" node. No Hamilton path or Hamilton circuit can be made when starting from the "Cafeteria" node. The reason for this can be caused by the following locations represented by blue and orange.

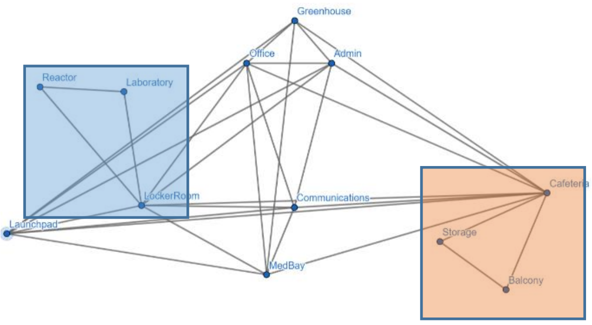

Figure 25. Graph Representation of Mira HQ with Colors for Possible Locations with Issue
Source: Writer

Note the nodes on the blue colored square. The nodes "Reactor" and "Laboratory" are only accessible through the "LockerRoom" node. Since the player's starting position is not on either of those nodes, if the players were to access "Reactor" or "Laboratory", they would need to have already accessed the "LockerRoom" node. This case is also true for the "Cafeteria", "Storage", and "Balcony" nodes. A player cannot access "Balcony" or "Storage" without first going into "Cafeteria". One of the ways to mitigate a case like this is to make those 3 nodes to be the last set of nodes for the player to visit. However, since there are two 3 sets of these nodes, it is impossible for the player to access both 3 sets last. Therefore, a Hamilton path or circuit is impossible to make from the game's starting position.

That said, similar to the previous case where The Skeld was analyzed, there is a Hamilton path for this graph. What one must do is consider the starting position as the one of the nodes that only have a degree of 2 in either the 3 sets of "Reactor", "Laboratory" and "LockerRoom" set, or the "Cafeteria", "Storage", and "Balcony" set. The following is an example using the "Reactor" node as a starting point and ending in the "Balcony" node.

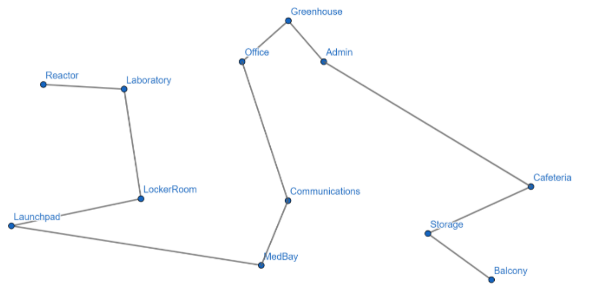

Figure 26. Hamilton Path of Mira HQ with Reactor as The Starting Node
Source: Writer

This shows that a Hamilton path is still available to access from the graph. When starting the game from the "LaunchPad" node, players can advance to the "Reactor" node and take this path in order to traverse all the key location exactly once. If the player were to start from the "Cafeteria" node, then the player can traverse firstly to the "Balcony" node and travel the Hamilton path in reverse.

4.3. Polus

The following is a graphical representation of the Polus map. Identical to the previous graphical representation, the nodes and edges also represents the key locations and possible routes that the player may take. There are 2 starting points for this graph. The first is the "Dropship" node and the second is the "Office" node. For this map, some nodes have the same name. The writer had written these locations deliberately with the same name and a number to identify the location. For the "Office" node, the node that will be the starting point is renamed in the graph as "Office$_1$".

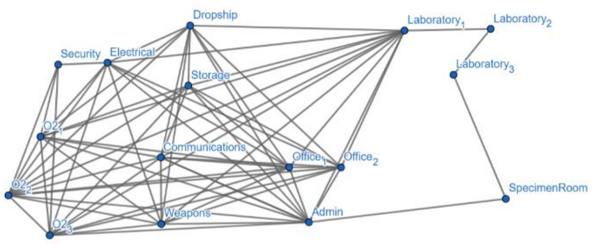

Figure 27. Graphical Representation of Polus
Source: Writer

Unlike the previous 2 maps, when considering the "Launchpad" and "Office$_1$" node as the starting position, a Hamilton circuit is accessible. Fortunately, there are many options to choose from. The following is one of the Hamilton circuit with respect to the "Launchpad" node as the starting position.

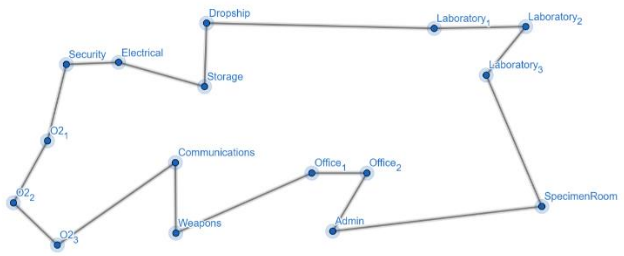

Figure 28. Hamilton Circuit of Polus with Dropship as a Starting Node
Source: Writer

When starting in the "Office$_1$" node, the player can also follow the same Hamilton circuit. This can be done because the circuit itself also incorporates the "Office$_1$" node in its path. However, to show that there are more than 1 possible Hamilton for this graph, consider the following circuit.

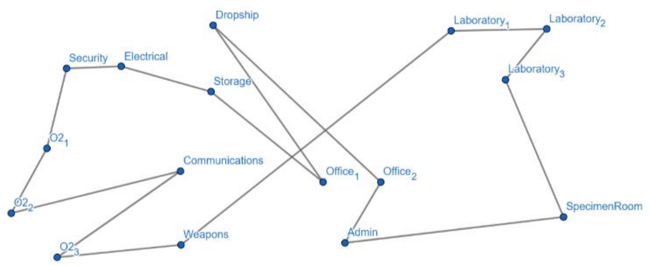

Figure 29. Another Example for a Hamilton Circuit in Polus
Source: Writer

As shown, this map has many options for a Hamilton circuit. This fact makes it easier for players to travel more efficiently in the Polus map. This is only the map where a Hamilton circuit is available for the player even if only considering the starting position given by the game. Therefore, for this map, whether the starting position is "Dropship" or "Office$_1$" there is always a Hamilton circuit to traverse upon.

V. Conclusion

To efficiently complete all task that are given in a game of Among Us, a method of travelling the map where a player only visits each of the possible location once needs to be established. This is done so that the player does not waste time travelling forward and backwards in the map that may cause more delay in the completion of tasks. One of the ways to achieve this is by using a Hamilton circuit or path. This way, no matter where the key location of the task is given, there is a sure way of visiting said location exactly once.

As shown in the analysis section, not all the maps in Among Us have Hamilton circuits or path when considering the starting position given by the game. This is shown in the analysis for The Skeld and Mira HQ. However, when one views the starting position as another node within the graph, a Hamilton path or circuit becomes available. This forces the player to move to the corresponding starting locations which have a possible Hamilton path/circuit. Although slightly inefficient due to having to first move to a desired location, this ensures that the previous benefits of travelling down a Hamilton path/circuit within the game is still accessible to the player.

As for the graph with a Hamilton circuit/path available to the player from the starting position of the game, it is easier for players to efficiently travel to each of the key location once. In this paper, the analysis has resulted in only Polus as the one map that contains a Hamilton circuit when considering only the starting position given by the game. This makes playing in Polus more favorable to crewmates because of the existence of a Hamilton circuit.

For a closing statement, this paper has found that there is a connection between graph theory, its application, and the map in Among Us. This paper is meant as a general study on what the application of discrete mathematics may be used for even for entertainment. There are still many improvements to be made upon this paper, such as the inclusion of more extensive game mechanic explanation, how the two different player types (the crewmate and impostor) would traverse the map differently, other ways or methods for player to traverse the map more efficiently, and many more. Overall, this paper's goals are to be a basis for much analysis and study. Hopefully, this paper can bring understanding and further be expanded upon.

VI. Acknowledgment

First and foremost, the writer of this paper would like to thank God for the opportunity of writing this paper. Secondly, the writer of this paper would like to acknowledge the teams of lecturers and professors in Bandung Institute of Technology for building a curriculum that allows this type of paper, albeit still with many flaws, to still be given as an assignment. The writer of this paper would like to give acknowledgement to Mrs. Fariska Zakhralativa R. as the lecturer who taught for class 3 in the Discrete Mathematics course and to Mr. Rinaldi Munir whose website, presentations, and lectures is a huge contributing factor to the completion of this paper. The writer would also like to acknowledge the help of friends and family members who have given the writer emotional support in writing this paper. Shoutout to Mrs. Nethania Alysia for her assistance in proofreading this paper. Lastly, the writer of this paper would like to thank the reader for reading this paper, hopefully it shed a little light on your understanding on how to apply discrete mathematics in basic things like an Among Us game.

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PERNYATAAN

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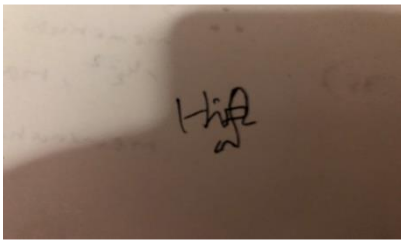
Bandung, 3 Desember 2020



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